Continuous Random Variables

Lecture 23
Section 7.5.4

Robb T. Koether

Hampden-Sydney College

Wed, Feb 29, 2012

Outline

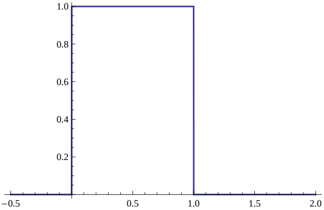
Outline

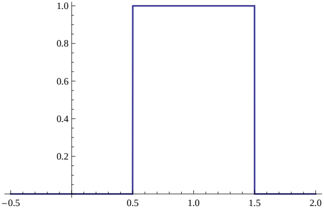
Outline

Hypothesis Testing ($n = 1$)

- An experiment is designed to determine whether a random variable X has the distribution $U(0, 1)$ or $U(0.5, 1.5)$.
  - $H_0 : X$ is $U(0, 1)$.
  - $H_1 : X$ is $U(0.5, 1.5)$.
- One value of X is sampled ($n = 1$).
- If X is more than 0.90, then H_0 will be rejected.

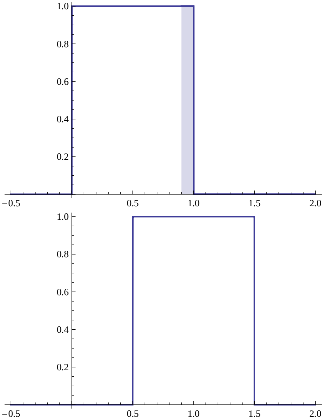
Hypothesis Testing ($n = 1$)





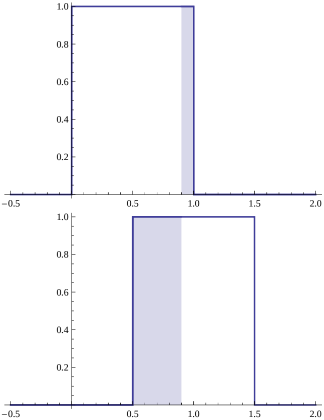
Hypothetical distributions of X under H_0 and H_1:

Hypothesis Testing ($n = 1$)



What is α?

Hypothesis Testing ($n = 1$)



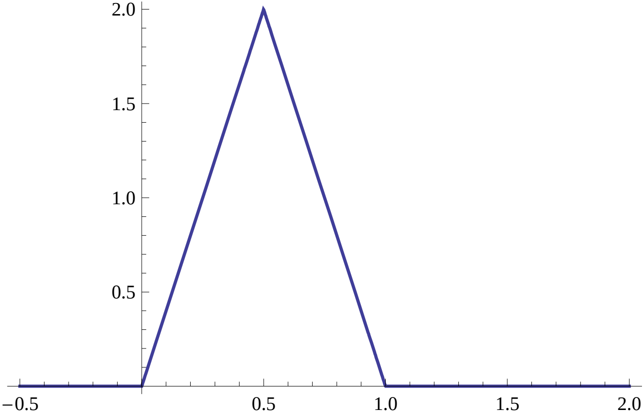
What is β?

Outline

Example

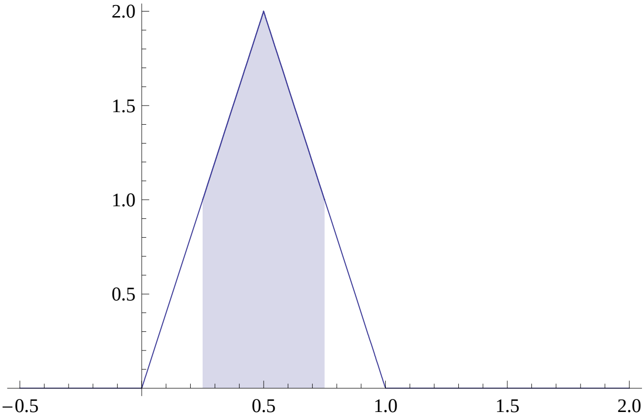
- Now suppose we use the TI-83 to get *two* random numbers from 0 to 1.
- Let $X_2 =$ the average of the two random numbers.
- What is the pdf of X_2?

Example



The graph of the pdf of X_2.

Example

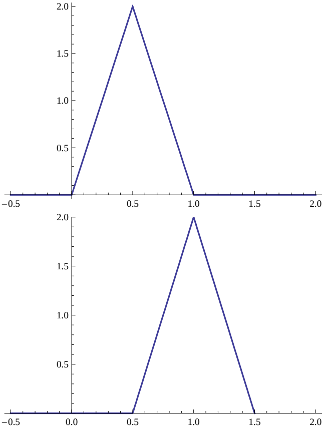


What is the probability that X_2 is between 0.25 and 0.75?

Hypothesis Testing ($n = 2$)

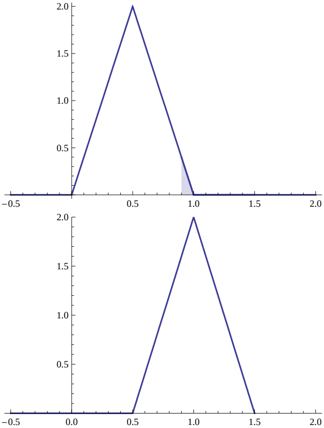
- An experiment is designed to determine whether a random variable X has the distribution $U(0, 1)$ or $U(0.5, 1.5)$.
  - $H_0 : X$ is $U(0, 1)$.
  - $H_1 : X$ is $U(0.5, 1.5)$.
- Two values of X are sampled ($n = 2$).
- Let X_2 be the average.
- If X_2 is more than 0.90, then H_0 will be rejected.

Hypothesis Testing ($n = 2$)

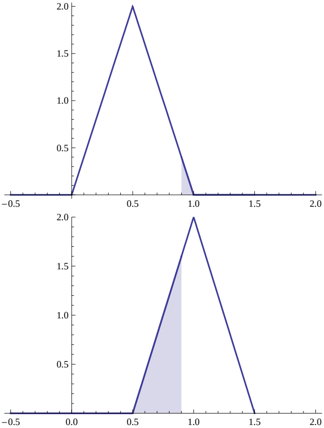


Hypothetical distributions of X under H_0 and H_1:

Hypothesis Testing ($n = 2$)



What is α?



What is β?

Conclusion

Conclusion

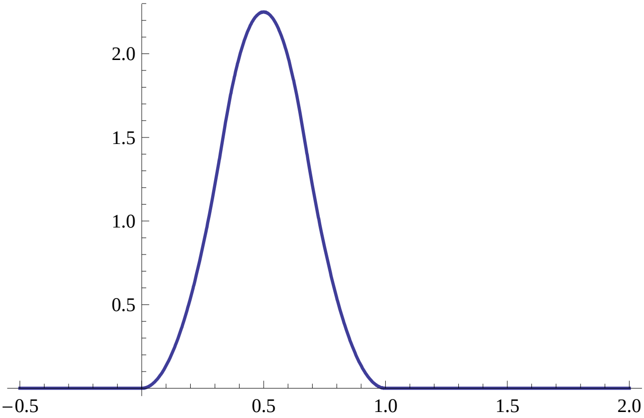
- By increasing the sample size, we can lower both α and β simultaneously.

Outline

Example

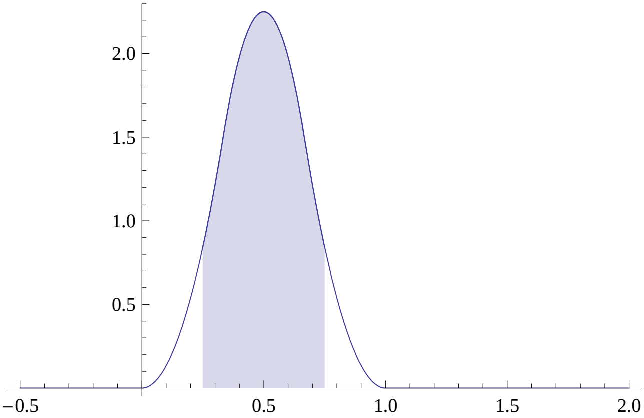
- Now suppose we use the TI-83 to get *three* random numbers from 0 to 1, and then average them.
- Let $X_3 =$ the average of the three random numbers.
- What is the pdf of X_3?

Example



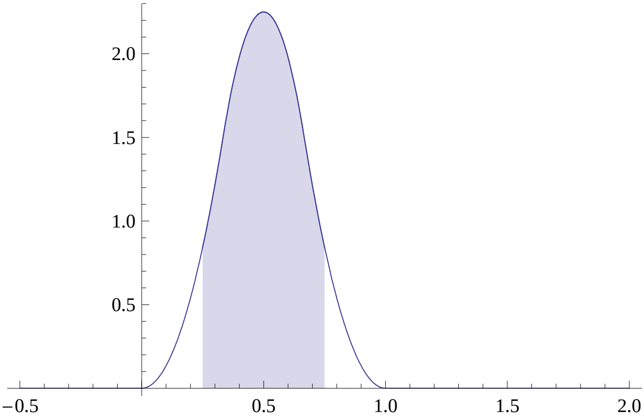
The graph of the pdf of X_3.

Example



What is the probability that X_3 is between 0.25 and 0.75?

Example

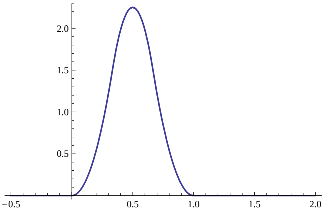


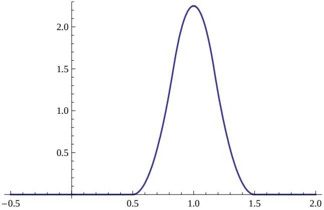
It turns out to be $\frac{55}{64} = 0.8954$.

Hypothesis Testing (n = 3)

- An experiment is designed to determine whether a random variable X has the distribution $U(0, 1)$ or $U(0.5, 1.5)$.
  - $H_0 : X$ is $U(0, 1)$.
  - $H_1 : X$ is $U(0.5, 1.5)$.
- Three values of X_3 are sampled ($n = 3$). Let X_3 be the average.
- If X_3 is more than 0.90, then H_0 will be rejected.

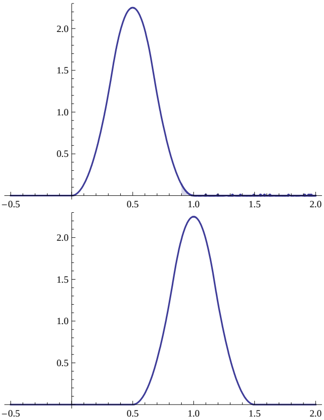
Hypothesis Testing (n = 3)





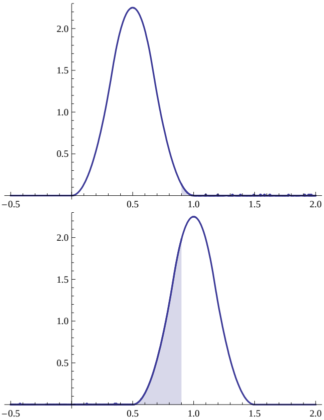
The hypothetical distributions of X_3 under H_0 and H_1.

Hypothesis Testing (n = 3)



It turns out that $\alpha = 0.0045$.

Hypothesis Testing (n = 3)



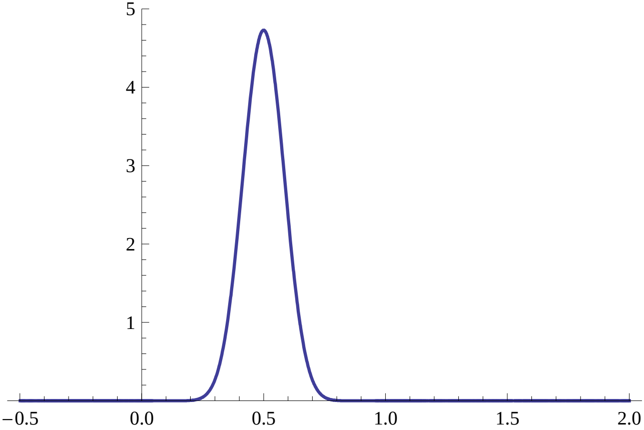
And $\beta = 0.2840$.

Outline

Example

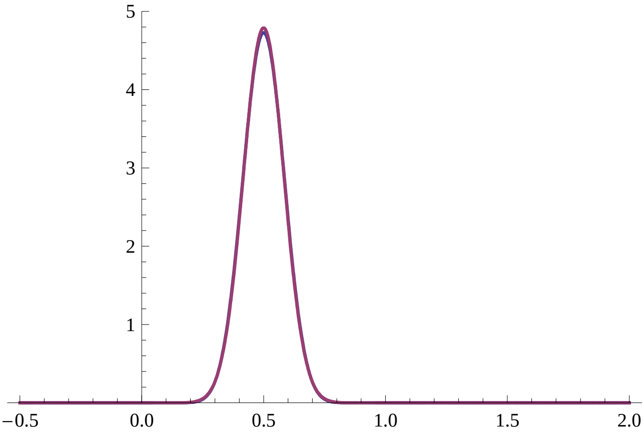
- Suppose we get 12 random numbers, uniformly distributed between 0 and 1, from the TI-83 and get their average.
- Let X_{12} = average of 12 random numbers from 0 to 1.
- What is the pdf of X_{12}?

Example



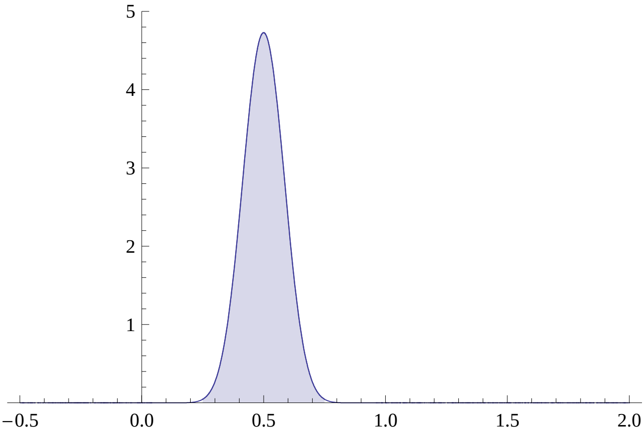
It turns out that the pdf of X_{12} is nearly exactly normal with a mean of $\frac{1}{2}$ and a standard deviation of $\frac{1}{12}$.

Example



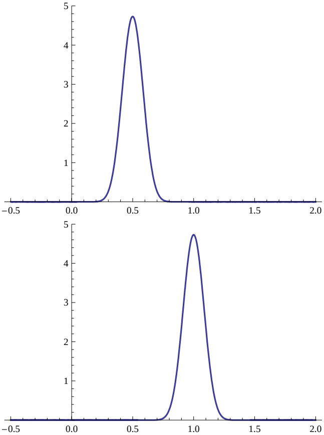
It turns out that the pdf of X_{12} is nearly exactly normal with a mean of $\frac{1}{2}$ and a standard deviation of $\frac{1}{12}$.

Example



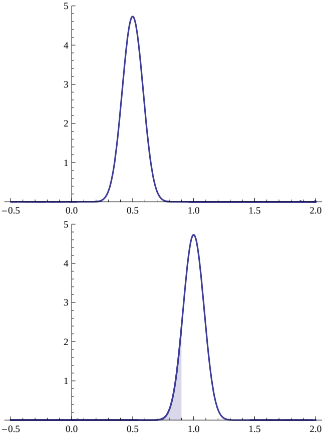
What is the probability that X_{12} is between 0.25 and 0.75?

Hypothesis Testing (n = 12)



The hypothetical distributions of X_{12} under H_0 and H_1.

Hypothesis Testing (n = 12)



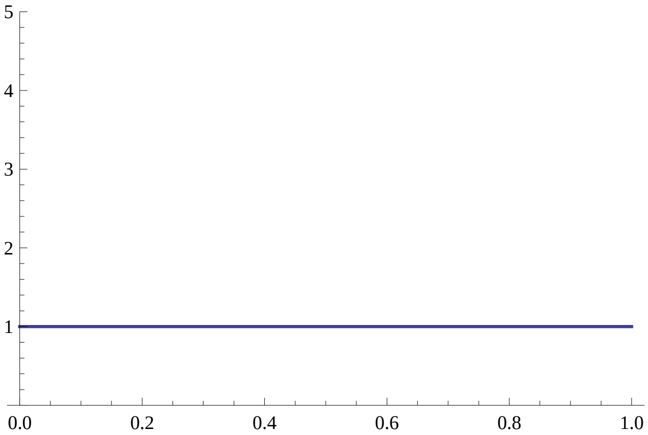
What are α and β?

Outline

Preview of the Central Limit Theorem

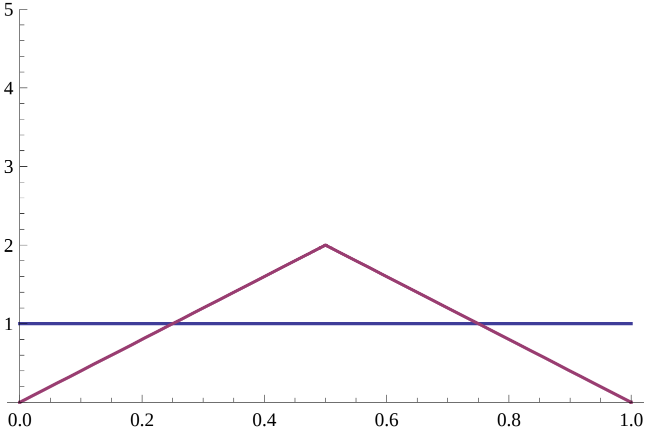
- We looked at the distribution of the average of 1, 2, 3, and 12 uniform random variables $U(0, 1)$.
- We saw that the shapes of their distributions was moving towards the shape of the normal distribution.

Preview of the Central Limit Theorem



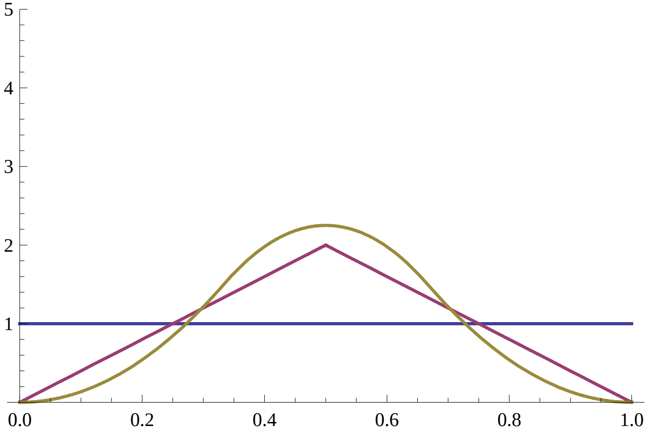
$n = 1$

Preview of the Central Limit Theorem



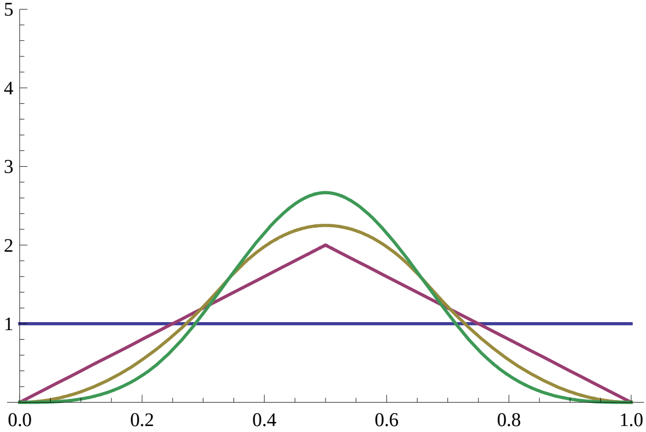
$n = 2$

Preview of the Central Limit Theorem



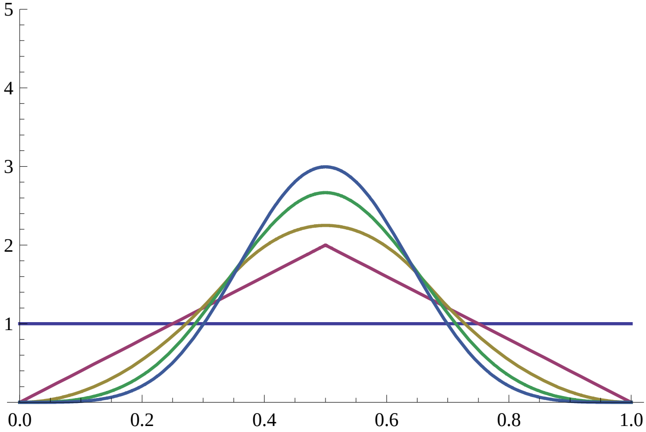
$n = 3$

Preview of the Central Limit Theorem



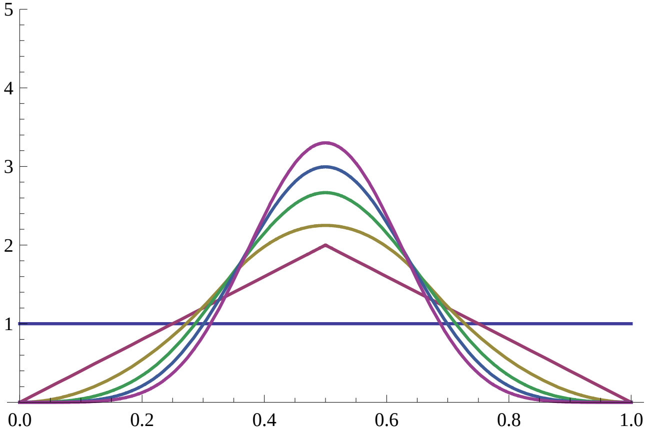
$n = 4$

Preview of the Central Limit Theorem



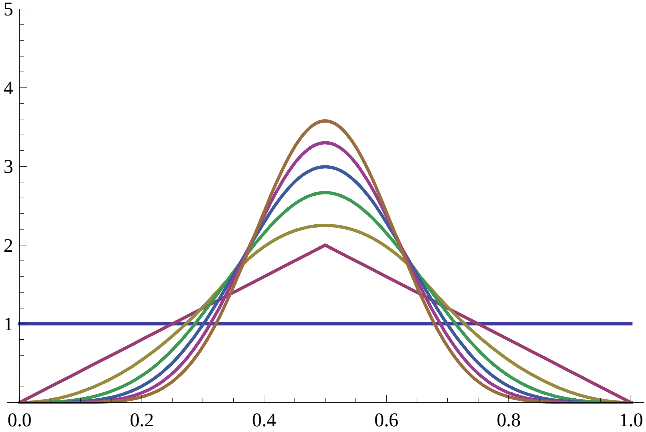
$$n = 5$$

Preview of the Central Limit Theorem



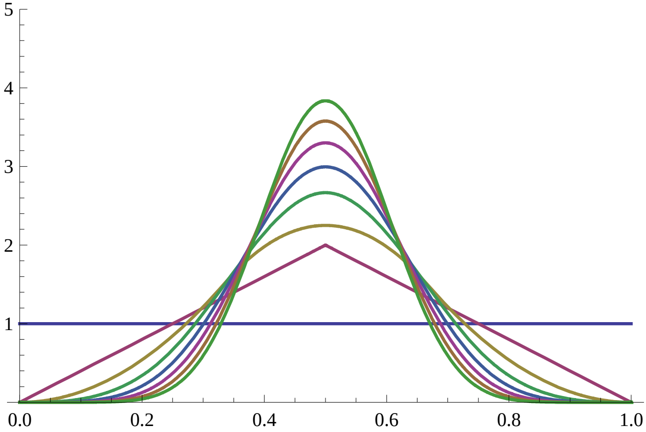
$n = 6$

Preview of the Central Limit Theorem



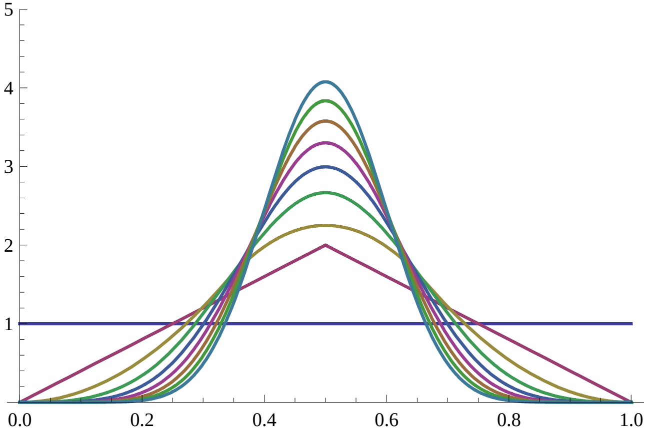
$n = 7$

Preview of the Central Limit Theorem



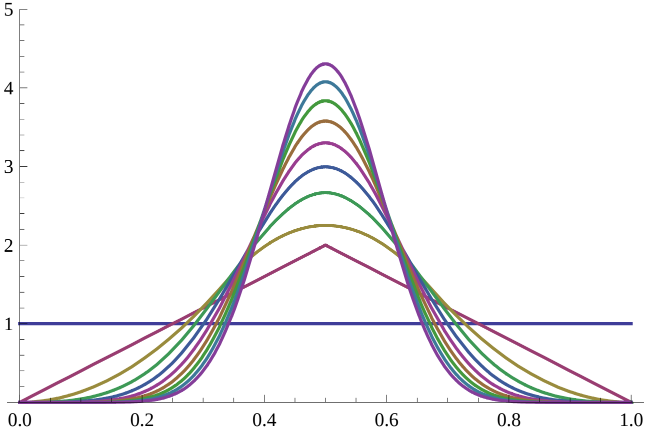
$n = 8$

Preview of the Central Limit Theorem



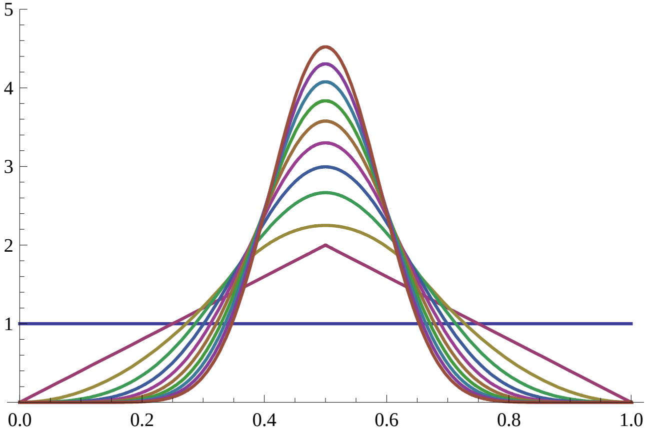
$n = 9$

Preview of the Central Limit Theorem



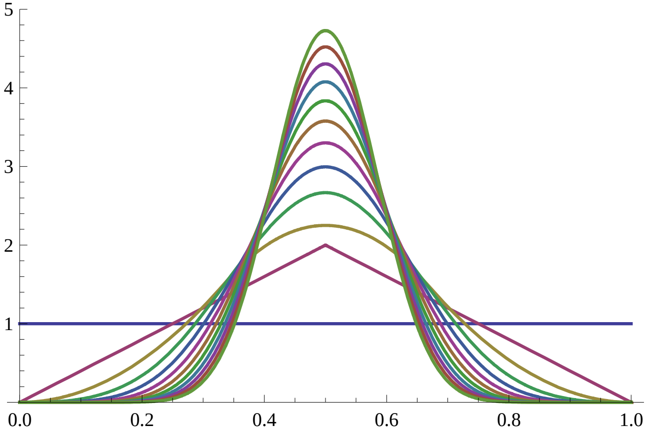
$n = 10$

Preview of the Central Limit Theorem



$n = 11$

Preview of the Central Limit Theorem



$n = 12$

Preview of the Central Limit Theorem

- Some observations:
  - Each distribution is centered at the same place, $\frac{1}{2}$.
  - The distributions are being "drawn in" towards the center.
  - That means that their standard deviations are decreasing as the sample size increases.
- Can we quantify this?

Outline

- Suppose that a population is 50% male and 50% female.

- Suppose that a population is 50% male and 50% female.
- We take a sample of 25 and compute the sample proportion of males.

- Suppose that a population is 50% male and 50% female.
- We take a sample of 25 and compute the sample proportion of males.
- The sample proportion could be anything from 0% to 100%, depending on the sample.

- Suppose that a population is 50% male and 50% female.
- We take a sample of 25 and compute the sample proportion of males.
- The sample proportion could be anything from 0% to 100%, depending on the sample.
- But it is probably close to 50%.

- Suppose that a population is 50% male and 50% female.
- We take a sample of 25 and compute the sample proportion of males.
- The sample proportion could be anything from 0% to 100%, depending on the sample.
- But it is probably close to 50%.
- We will simulate this situation.

- Use `rand(0,1,25)` to randomly select twenty-five 0's and 1's.

- Use `rand(0,1,25)` to randomly select twenty-five 0's and 1's.
- Interpret 1 as male, 0 as female.

- Use `rand(0,1,25)` to randomly select twenty-five 0's and 1's.
- Interpret 1 as male, 0 as female.
- Compute the sample proportion of males.

- Use `rand(0,1,25)` to randomly select twenty-five 0's and 1's.
- Interpret 1 as male, 0 as female.
- Compute the sample proportion of males.
- Then apply the `sum` function to count the 1's.

- Use `rand(0,1,25)` to randomly select twenty-five 0's and 1's.
- Interpret 1 as male, 0 as female.
- Compute the sample proportion of males.
- Then apply the `sum` function to count the 1's.
- A simpler way to do this is to enter `randBin(25,.5)`.

- Use `rand(0,1,25)` to randomly select twenty-five 0's and 1's.
- Interpret 1 as male, 0 as female.
- Compute the sample proportion of males.
- Then apply the `sum` function to count the 1's.
- A simpler way to do this is to enter `randBin(25,.5)`.
- Repeat this many times until we can see the shape of the distribution.

Outline

Assignment

Homework

- Read Sections 8.1, 8.2.
- Work Example 8.2 as homework.
- Work Let's Do It! 8.1, 8.2 as homework.